ZADACI ZA VJEŽBU - INTEGRALI

1. Provjeri je li F primitivna funkcija funkciji f ako je zadano:

a) $F(x) = \sin x + C,\ C \in R, \quad f(x) = \cos x;$

b) $F(x) = \dfrac{x^8}{2} + C,\ C \in R, \quad f(x) = 4x^7.$

2. Direktnim integriranjem riješi:

a) $\displaystyle\int (2x\sqrt{x} - 13)\ dx$

b) $\displaystyle\int (5x^4 + 12x^3 - 2x + 6)\ dx$

c) $\displaystyle\int \left(\frac{3}{x} - e^x + x^{-2}\right)\ dx.$

3. Metodom supstitucije integriraj:

a) $\displaystyle\int \sqrt[3]{5 - 4x}\ dx$

b) $\displaystyle\int \left(2 - \frac{1}{9}\right)^{-10}\ dx$

c) $\displaystyle\int \frac{x}{x^2 + 3}\ dx$

d) $\displaystyle\int \frac{2e^{2x}}{(e^{2x} + 1)^2}\ dx.$

4. Metodom parcijalne integracije integriraj:

a) $\displaystyle\int \sqrt{x} \ln x\ dx$

b) $\displaystyle\int \frac{x^2}{3}\ e^x\ dx$

c) $\displaystyle\int x^2 \cos 2x\ dx.$

5. Riješi sljedeće integrale racionalnih funkcija:

a) $\displaystyle\int \frac{2}{5x-3}\,dx$

b) $\displaystyle\int \frac{2x^3}{x-3}\,dx$

c) $\displaystyle\int \frac{1}{(x-3)(1+2x)}\,dx$

d) $\displaystyle\int \frac{2x+1}{x^2+x+1}\,dx$

e) $\displaystyle\int \frac{1}{x^2+9}\,dx$

f) $\displaystyle\int \frac{x+1}{x^3+x}\,dx.$

6. Riješi sljedeće integrale trigonometrijskih funkcija:

a) $\displaystyle\int \frac{dx}{1+\sin x+\cos x}$

b) $\displaystyle\int \cos^2 x\ dx$

c) $\displaystyle\int \sin^{10} x \cos^3 x\ dx$

d) $\displaystyle\int \frac{\cos^2 x}{\sin^6 x}\,dx$

e) $\displaystyle\int \frac{\cos x}{1+\cos x}\,dx.$

RJEŠENJA

2. a) $\frac{4}{5}x^{\frac{5}{2}} - 13x + C$; b) $x^5 + 3x^4 - x^2 + 6x + C$; c) $3\ln|x| - e^x - \frac{1}{x} + C$.

3. a) $-\frac{3}{16}(5-4x)^{\frac{4}{3}} + C$; b) $(2 - \frac{1}{9}x)^{-9} + C$; c) $\frac{1}{2}\ln(x^2+3) + C$; d) $-\frac{1}{e^{2x}+1} + C$.

4. a) $-\frac{4x\sqrt{x}}{9} + \frac{2x\sqrt{x}\ln x}{3} + C$; b) $\frac{1}{3}x^2e^x - \frac{2}{3}xe^x + \frac{2}{3}e^x + C$; c) $\frac{1}{2}x^2\sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + C$.

5. a) $\frac{2}{5}\ln|5x-3| + C$; b) $\frac{2x^3}{3} + 3x^2 + 18x + 54\ln|x-3| + C$; c) $\frac{1}{7}\ln|x-3| - \frac{1}{7}\ln|1+2x| + C$;
d) $\ln|x^2+x+1| + C$; e) $\frac{1}{3}\operatorname{arctg}\frac{x}{3} + C$; f) $\ln|x| - \frac{1}{2}\ln(x^2+1) + \operatorname{arctg} x + C$.

6. a) $\ln|1 + \operatorname{tg}\frac{x}{2}| + C$; b) $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$; c) $\frac{\sin^{11} x}{11} - \frac{\sin^{13} x}{13} + C$; d) $-\frac{\operatorname{ctg}^3 x}{3} - \frac{\operatorname{ctg}^5 x}{5} + C$;
e) $x - \operatorname{tg}\frac{x}{2} + C$.